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Engineering Notes

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Nonlinear Attitude State Tracking Control for Spacecraft

Guang Q. Xing* and Shabbir A. Parvez†
Space Products and Applications, Inc.,
Fairfax, Virginia 22030

Introduction

SPACECRAFT attitude control for large-angle slew maneuvers involves nonlinear governing equations, along with modeling uncertainty and unexpected external disturbances. Existing literature indicates two possible control design options for overcoming these difficulties: 1) Lyapunov nonlinear controller[1–4] and 2) nonlinear robust controller using sliding control method.[3,5,6] Most of the control systems provided in the references are position control only. In this paper, however, this problem is being revisited through application of relative attitude kinematics and dynamics equations that have been developed in Ref. 7. The advantage of using relative attitude kinematics and dynamics equations to construct control law is that it can convert the tracking-control problem into regulator problem. This simplifies the design problem because a regulator design is easier than a tracking-control design. In addition, the controller designed using this method implements both position and velocity tracking. Thus, it presents a general framework for solving attitude state tracking (position and velocity tracking) control for large-angle maneuvers of spacecraft.

Lyapunov Attitude State Tracking Control

To avoid singular points, the modified Rodrigues parameters (MRP) p are selected to describe the orientation of the satellite attitude. It is assumed that p_b, ω_b and p_d, ω_d are the absolute attitudes and angular velocities of the chase and target satellites, respectively. The problem is to find a control law that can transfer the state (p_b, ω_b) of the chase satellite to be the state (p_d, ω_d) of the target satellite.

It is assumed that the relative attitude for the modified Rodrigues parameter is defined as follows[7]:

$$p_{\mathrm{bd}} = p_b \otimes p_d^{-1} = \frac{p_d\left(p_b^T p_b - 1\right) + p_b\left(1 - p_d^T p_d\right) - 2p_d^{\times} p_b}{1 + p_d^T p_d p_b^T p_b + 2p_d^T p_d} \tag{1}$$

and the relative angular velocity is defined as

$$\omega_{\mathrm{bd}} = \omega_b - R_{\mathrm{BD}}\omega_d \tag{2}$$

where R_{BD} is the coordinate transformation matrix from the body reference system of target spacecraft to that of chase spacecraft. The attitude state of this system is relative attitude and relative angular velocity (p_{bd}, ω_{bd}). The tracking goal is to find a control law to transfer the relative state (p_{bd}, ω_{bd}) of chase satellite to be $(0,0) \in \mathbb{R}$.[6] Thus the tracking problem is converted into a regulator problem, and the design of regulator control is much simpler than the design of tracking control. The Lyapunov method can be applied directly to develop the nonlinear control law.

The relative attitude kinematics equation for MRP is[7]

$$\dot{p}_{\mathrm{bd}} = M\omega_{\mathrm{bd}} \tag{3}$$

where

$$M = \tfrac{1}{4}\left\{\left(1 - p_{\mathrm{bd}}^T p_{\mathrm{bd}}\right)I + 2\left[p_{\mathrm{bd}}^{\times}\right] + 2p_{\mathrm{bd}}p_{\mathrm{bd}}^T\right\} \tag{4}$$

$$M^{-1} = \left[4\Big/\left(1 + p_{\mathrm{bd}}^T p_{\mathrm{bd}}\right)^2\right]\left\{\left(1 - p_{\mathrm{bd}}^T p_{\mathrm{bd}}\right) + 2p_{\mathrm{bd}}p_{\mathrm{bd}}^T - 2\left[p_{\mathrm{bd}}^{\times}\right]\right\} \tag{5}$$

The relative attitude dynamics equation in chase spacecraft body reference system is[7]

$$J_b\dot{\omega}_{\mathrm{bd}} + \left[\omega_{\mathrm{bd}}^{\times}\right]J_b\omega_{\mathrm{bd}} + \left[\omega_{\mathrm{bd}}^{\times}\right]J_b R_{\mathrm{BD}}\omega_d = L \tag{6}$$

where

$$\begin{aligned} L = {} & L_b - R_{\mathrm{BD}}L_d - \left[(R_B R\omega_d)^{\times}\right]J_b\omega_{\mathrm{bd}} + J_b\left[\omega_{\mathrm{bd}}^{\times}\right]R_{\mathrm{BD}}\omega_d \\ & - R_{\mathrm{BD}}\left(\Delta J_d\dot{\omega}_d + \left[\omega_d^{\times}\right]\Delta J_d\omega_d\right) \end{aligned} \tag{7}$$

$$\begin{aligned} R_{\mathrm{BD}} = {} & \left[1\Big/\left(1 + p_{\mathrm{bd}}^T p_{\mathrm{bd}}\right)^2\right]\left\{\left[1 - 6p_{\mathrm{bd}}^T p_{\mathrm{bd}} + \left(p_{\mathrm{bd}}^T p_{\mathrm{bd}}\right)^2\right]I \right. \\ & \left. + 8p_{\mathrm{bd}}p_{\mathrm{bd}}^T - 4\left(1 - p_{\mathrm{bd}}^T p_{\mathrm{bd}}\right)\left[p_{\mathrm{bd}}^{\times}\right]\right\} \end{aligned} \tag{8}$$

$$\Delta J_d = R_{\mathrm{BD}}^T J_b R_{\mathrm{BD}} - J_d \tag{9}$$

If the relative attitude p_{bd} and relative velocity ω_{bd} are selected to be the attitude state vector, then Eqs. (3) and (6) are the relative attitude state equations.

Let the state vector of this system be defined as

$$x \equiv \begin{bmatrix}\Delta p \\ \Delta\omega\end{bmatrix} \equiv \begin{bmatrix}p_{\mathrm{bd}} \\ \omega_{\mathrm{bd}}\end{bmatrix} \tag{10}$$

The Lyapunov function V can be selected to be

$$V = \tfrac{1}{2}\left(K_p\Delta p^T\Delta p + \Delta\omega^T J_b\Delta\omega\right) \tag{11}$$

where K_p is a positive constant. The time derivative of Lyapunov function V is

$$\begin{aligned} \dot{V} & = K_p\Delta p^T\Delta\dot{p} + \Delta\omega^T J_b\Delta\dot{\omega} = K_p\Delta\omega^T M^T\Delta p \\ & \quad + \Delta\omega^T J_b\Delta\dot{\omega} = \Delta\omega^T\left(K_p M^T\Delta p + J_b\Delta\dot{\omega}\right) \\ & = \Delta\omega^T\left(K_p M^T\Delta p + L - [\Delta\omega^{\times}]J_b\Delta\omega \right. \\ & \quad \left. - [\Delta\omega^{\times}]J_b R_{\mathrm{BD}}\omega_d\right) = \Delta\omega^T\left(K_p M^T\Delta p + L\right) \end{aligned} \tag{12}$$

Now, if

$$L = -K_p M^T\Delta p - K_d J_b\Delta\omega \tag{13}$$

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*Principal Scientist; xingspa@erols.com.
†President; parvzspa@erols.com. Associate Fellow AIAA.

then

$$\dot{V} = -K_d \Delta\omega^T J_b \Delta\omega \leq 0 \tag{14}$$

where K_d is a positive constant. The closed-loop system dynamics equation can be obtained from Eq. (6):

$$J_b \Delta\dot{\omega} + [\Delta\omega^\times] J_b \Delta\omega + [\Delta\omega^\times] J_b R_{\mathrm{BD}} \omega_d$$
$$= -K_p M^T \Delta p - K_d J_b \Delta\omega \tag{15}$$

Equation (14) implies that $V(t) \leq V(0)$, and therefore Δp and $\Delta\omega$ are bounded. In addition, the second-order derivative of Lyapunov V can be calculated from Eq. (14) as

$$\ddot{V} = -2K_d \Delta\omega^T J_b \Delta\dot{\omega} \tag{16}$$

It can be seen from Eq. (15) that the second-order derivative of V is bounded because Δp and $\Delta\omega$ were shown to be bounded. Hence $\mathrm{d}V/\mathrm{d}t$ is uniformly continuous.[8] Application of Barbalat's lemma[8] then indicates that $\Delta\omega \to 0$ as $t \to \infty$. Considering the closed-loop equation (15), it can be shown that $\Delta p \to 0$ as $t \to \infty$. Therefore, it can be shown that $(\Delta p, \Delta\omega) \to (0.0) \in \mathbb{R}^6$ as $t \to \infty$. It implies that the Lyapunov nonlinear control law

$$L_b = -K_p M^T \Delta p - K_d J_b \Delta\omega + R_{\mathrm{BD}} L_d + \left[(R_{\mathrm{BR}}\omega_d)^\times\right] J_b \Delta\omega$$
$$- J_b [\Delta\omega^\times] R_{\mathrm{BD}} \omega_d + R_{\mathrm{BD}}\left(\Delta J_d \dot{\omega}_d - \left[\omega_d^\times\right] \Delta J_d \omega_d\right) \tag{17}$$

is a global asymptotically stable control law for the system given by Eqs. (3) and (6). Equation (17) is obtained from Eqs. (7) and (13).

Sliding Robust Nonlinear Controller

Substituting the kinematics equation (3) into the dynamics equation (6) and rearranging yields

$$J \ddot{p}_{\mathrm{bd}} + C \dot{p}_{\mathrm{bd}} = (M^{-1})^T L \tag{18}$$

$$C = M^{-T} J_b \frac{\mathrm{d}M^{-1}}{\mathrm{d}t} - (M^{-1})^T \left[(J_b \omega_{\mathrm{bd}})^\times\right] M^{-1}$$
$$- M^{-T}\left[(J_b R_{\mathrm{BD}} \omega_d)^\times\right] M^{-1} \tag{19}$$

where

$$J = (M^{-1})^T J_b M^{-1} \tag{20}$$

$$L = L_b - R_{\mathrm{BD}} L_d - \left[(R_{\mathrm{BR}}\omega_d)^\times\right] J_b \omega_{\mathrm{bd}} + J_b \left[\omega_{\mathrm{bd}}^\times\right] R_{\mathrm{BD}} \omega_d$$
$$- R_{\mathrm{BD}}\left(\Delta J_d \dot{\omega}_d + \left[\omega_d^\times\right] \Delta J_d \omega_d\right) \tag{21}$$

The attitude state vector of the system be defined as

$$x \equiv \begin{bmatrix} \Delta x \\ \Delta\dot{x} \end{bmatrix} \equiv \begin{bmatrix} p_{\mathrm{bd}} \\ \dot{p}_{\mathrm{bd}} \end{bmatrix} \tag{22}$$

and the dynamics equation (18) can be written as

$$J \Delta\ddot{x} + C \Delta\dot{x} = (M^{-1})^T L \tag{23}$$

The derivation of a robust sliding attitude state controller can now be considered. For multi-input cases the sliding condition can be written as the vector form[8]

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t} s^T s \leq -\eta (s^T s)^{\frac{1}{2}}, \qquad (\eta > 0) \tag{24}$$

The vector s is defined as

$$s = \Delta\dot{x} + \Lambda \Delta x \tag{25}$$

where Λ is a symmetric positive definite matrix, a matrix where $-\Lambda$ is Hurwitz. The vector s conveys information about boundedness and convergence of Δx and $\mathrm{d}\Delta x/\mathrm{d}t$ because the definition (25) of s can also be viewed as a stable first-order differential equation in Δx, with s as an input. Thus, assuming bounded initial conditions, proving the boundedness of s also implies the boundedness of Δx and $\mathrm{d}\Delta x/\mathrm{d}t$ and therefore of x and $\mathrm{d}x/\mathrm{d}t$; similarly, if s tends to 0 as t tends to infinity, so do the vectors Δx and $\mathrm{d}\Delta x/\mathrm{d}t$.

Defining

$$V = \tfrac{1}{2}[s^T J s] \tag{26}$$

and differentiating

$$\dot{V}(t) = s^T J \dot{s} + \tfrac{1}{2} s^T \dot{J} s = s^T \left(J\dot{s} + \tfrac{1}{2}\dot{J} s\right) \tag{27}$$

Because

$$\dot{s} = \Delta\ddot{x} + \Lambda \Delta\dot{x} \tag{28}$$

therefore

$$\dot{V}(t) = s^T\left[J(\Delta\ddot{x} + \Lambda\Delta\dot{x}) + \tfrac{1}{2}\dot{J}s\right] = s^T\left[(M^{-T}L - C\Delta\dot{x})\right.$$
$$\left. + J\Lambda\Delta\dot{x} + \tfrac{1}{2}\dot{J}s\right] = s^T\left[M^{-T}L - C(s - \Lambda\Delta x) + J\Lambda\Delta\dot{x}\right.$$
$$\left. + \tfrac{1}{2}\dot{J}s\right] = s^T\left[M^{-T}L + \tfrac{1}{2}(\dot{J} - 2C)s + J\Lambda\Delta\dot{x} + C\Lambda\Delta x\right] \tag{29}$$

Because

$$\dot{J} - 2C = \frac{\mathrm{d}M^{-T}}{\mathrm{d}t} J_b M^{-1} + M^{-T} J_b \frac{\mathrm{d}M^{-1}}{\mathrm{d}t} - 2\left\{M^{-T} J_b \frac{\mathrm{d}M^{-1}}{\mathrm{d}t}\right.$$
$$\left. - M^{-T}\left[(J_b\Delta\omega)^\times\right]M^{-1} - M^{-T}\left[(J_b R_{\mathrm{BD}}\omega_d)^\times\right]M^{-1}\right\}$$
$$= \frac{\mathrm{d}M^{-T}}{\mathrm{d}t} J_b M^{-1} - M^{-T} J_b \frac{\mathrm{d}M^{-1}}{\mathrm{d}t} + 2M^{-T}\left[(J_b\Delta\omega)^\times\right]M^{-1}$$
$$+ 2M^{-T}\left[(J_b R_{\mathrm{BD}}\omega_d)^\times\right]M^{-1} \tag{30}$$

it is evident that the $\mathrm{d}J/\mathrm{d}t - 2C$ is a skew-symmetric matrix. Therefore, Eq. (29) will be reduced to

$$\dot{V}(t) = s^T[M^{-T}L + J\Lambda\Delta\dot{x} + C\Lambda\Delta x] \tag{31}$$

Let

$$M^{-T}L = \hat{L} - k\,\mathrm{sgn}(s) \tag{32}$$

$$\hat{L} = -\hat{J}\Lambda\Delta\dot{x} - \hat{C}\Lambda\Delta x \tag{33}$$

where $k\,\mathrm{sgn}(s)$ is defined as the vector of component $k_i\,\mathrm{sgn}(s_i)$. Then

$$\dot{V} = s^T[\hat{L} - K\,\mathrm{sgn}(s) + J\Lambda\Delta\dot{x} + C\Lambda\Delta x]$$
$$= s^T[-\Delta J\Lambda\Delta\dot{x} - \Delta C\Lambda\Delta x] - \sum_1^n K_i|s_i| \tag{34}$$

$$\mathrm{sgn}(s) = \begin{cases} 1 & s > 0 \\ 1 & s < 0 \end{cases} \tag{35}$$

where

$$\hat{J} - J = \Delta J, \qquad \hat{C} - C = \Delta C \tag{36}$$

With the requirement

$$\dot{V} \leq -\sum_1^n \eta_i |s_i| \tag{37}$$

that is,

$$\sum_1^n K_i|s_i| \geq \sum_1^n \eta_i|s_i| + s^T[\Delta J\Lambda\Delta\dot{x} + \Delta C\Lambda\Delta x] \tag{38}$$

sufficient condition for satisfying Eq. (38) will be

$$K_i \geq |[\Delta J\Lambda\Delta\dot{x} + \Delta C\Lambda\Delta x]_i| + \eta_i \tag{39}$$

Finally, considering Eqs. (21) and (33), the nonlinear sliding robust attitude control law becomes

$$L_b = M^T(\hat{J}\wedge\Delta\dot{x} + \hat{C}\wedge\Delta x) - M^T k\,\mathrm{sgn}(s) + R_{\mathrm{BD}}L_d + \left[(R_{\mathrm{BD}}\omega_d)^\times\right]J_b\omega_{\mathrm{bd}} - J_b\left[\omega_{\mathrm{bd}}^\times\right]R_{\mathrm{BD}}\omega_d + R_{\mathrm{BD}}\left(\Delta J_d\dot{\omega}_d + \left[\omega_d^\times\right]\Delta J_d\omega_d\right) \tag{40}$$

If the selection of parameters k_i satisfy the condition (39), then control law given by Eq. (40) will satisfy the sliding condition (37) and thus lead to "perfect" tracking in the face of model uncertainty. However, it is discontinuous across the surface $s(t)$ and will result in control chattering. Chattering is undesirable because it involves extremely high control activity and may excite high-frequency dynamics that were neglected in the course of modeling. Elimination of the chattering through modification of the switching control law just derived has been discussed in Refs. 8 and 9. Based on these references, continuous approximations of the switching control law, the sgn(.) switching function, is replaced by the sat(.) nonlinear saturation function. The effect of control interpolation in the boundary layer points to assigning low-pass filter structure to the local dynamics of the variable s, thus eliminating chattering.[8,9]

Conclusions

Two attitude state controllers have been designed for tracking of large-angle maneuvers by applying a set of relative attitude kinematics and dynamics equations, where the attitude is represented by the relative modified Rodrigues parameters. The designs are the global asymptotically stable nonlinear Lyapunov controller and the robust sliding controller for attitude state tracking. Using relative attitude state equations to design tracking controller converts the tracking control problem into a regulator problem and simplifies the design procedure. The controllers implement both attitude position and angular velocity tracking. The structure of this controller can also be used for other relative attitude parameters, such as Rodrigues parameters, Euler angles, and so on. It can provide a general solution to state tracking control for rigid-body attitude.

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References

[1]Wie, B., and Barba, P. M., "Quaternion Feedback for Spacecraft Large Angle Maneuvers," *Journal of Guidance, Control, and Dynamics*, Vol. 8, No. 3, 1985, pp. 360–365.

[2]Wen, J. T-Y., and Kreutz-Delgado, K. K., "The Attitude Control Problem," *IEEE Transactions on Automatic Control*, Vol. 36, No. 10, 1991, pp. 1148–1162.

[3]Crassidis, J. L., and Markley, F. L., "Sliding Mode Control Using Modified Rodrigues Parameters," *Journal of Guidance, Control, and Dynamics*, Vol. 19, No. 6, 1996, pp. 1381–1383.

[4]Hall, C. D., Tsiotras, P., and Shen, H., "Tracking Rigid Body Motion Using Thrusters and Momentum Wheels," AIAA Paper 98-4471, Aug. 1998.

[5]Vadali, S. R., "Variable-Structure Control of Spacecraft Large-Angle Maneuvers," *Journal of Guidance, Control, and Dynamics*, Vol. 9, No. 2, 1986, pp. 235–239.

[6]Dwyer, T. A. W., and Ramirez, H. S., "Variable-Structure Control of Spacecraft Attitude Maneuvers," *Journal of Guidance, Control, and Dynamics*, Vol. 11, No. 3, 1988, pp. 262–269.

[7]Xing, G. Q., and Parvez, S. A., "Relative Attitude Kinematics and Dynamics Equation and Its Application to Large Angle Maneuvers and Tracking," *Proceeding of 1999 Space Control Conference*, edited by L. B. Spence, 1999, MIT Lincoln Lab., Cambridge, MA, pp. 105–114.

[8]Slotine, J. E., and Li, W., *Applied Nonlinear Control*, Prentice–Hall, Upper Saddle River, NJ, 1991, pp. 122–126.

[9]Kim, J., Kim, J., and Crassidis, J. L., "Disturbance Accommodating Sliding Mode Controller for Spacecraft Attitude Maneuvers," *Proceedings of the AAS/GSFC 13th International Symposium on Space Flight Dynamics*, Vol. 1, edited by Tom Stengle, NASA, Greenbelt, MD, 1998, pp. 119–131.

Optimal Low-Thrust Earth–Moon Targeting Strategy for *N*-Body Problem

Hui Yan* and Hongxin Wu†
Chinese Academy of Space Technology,
100080 Beijing, People's Republic of China

Introduction

THERE are many papers on determining minimum-fuel, low-thrust, Earth–moon trajectories. With regard to recent research, Kluever and Pierson[1] and Herman and Conway[2] studied optimal low-thrust, three-dimensional Earth–moon trajectories for restricted three-body problems. Their solutions, like patched conic orbits with impulsive maneuvers, are good approximations to the n-body problem. The question is how to obtain accurate solutions based on the approximations for the n-body problem. This Note presents a targeting method to adjust approximate low-thrust Earth–moon trajectories to satisfy the given requirements. We take an optimal low-thrust trajectory in the Earth's gravity field as a reference trajectory. Based on this, the optimal low-thrust Earth–moon trajectory can be obtained using a differential correction algorithm.

Optimal Low-Thrust Trajectory for Two-Body Problem

As is well known, Earth–moon trajectories are easily established by the impulsive maneuver for the n-body problem. When the trajectory is set up for the prescribed targeting parameters, the osculating ellipse at the perigee can be obtained with classical orbital elements of a_e, e_e, i_e, Ω_e, ω_e, and T_e, the semimajor axis, eccentricity, inclination, longitude of the ascending node, argument of periapsis, and time of periapsis passage, respectively. If we make the spacecraft enter the osculating elliptical orbit from the low Earth parking orbit using the optimal low-thrust vector in the single gravity field of the Earth, it is expected that we should obtain an approximate or reference optimal thrust Earth–moon orbit transfer under the control of the thrust in the n-body problem. The equations for the reference trajectory will be established in perifocal coordinates. Here the fundamental plane is coplanar with the osculating ellipse. The origin is located at Earth's center. The coordinate axes are X_ω, Y_ω, and Z_ω. The X_ω axis points toward the periapsis of the osculating ellipse; the Y_ω axis is rotated 90 deg in the direction of the elliptical motion and lies in the fundamental plane; the Z_ω axis completes the right-handed perifocal system. Let the states for the equations be $x = (r, \theta, v_r, v_\theta)^T$. They are the radial position, polar angle, radial velocity, and circumferential velocity, respectively. The equations and initial conditions are

$$\dot{r} = v_r \qquad r(0) = r_{\mathrm{LEO}} \tag{1}$$

$$\dot{\theta} = v_\theta / r \qquad \theta(0) = 0 \tag{2}$$

$$\dot{v}_r = \left(v_\theta^2 \big/ r\right) - \mu/r^2 + a_P \sin u \qquad v_r(0) = 0 \tag{3}$$

$$\dot{v}_\theta = -(v_r v_\theta / r) + a_P \cos u \qquad v_\theta(0) = \sqrt{\mu / r_{\mathrm{LEO}}} \tag{4}$$

where

$$a_P = P/(m_0 - \dot{m}t) \tag{5}$$

and the thrust angle u is the control variable, P is the thrust magnitude, a_P is the thrust acceleration, μ is the gravitational constant, m_0 is the initial spacecraft mass, $\dot{m}$ is the propellant mass flow rate, r_{LEO} is the radius of low Earth park orbit, and t is time.

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*Postdoctoral Researcher, Beijing Institute of Control and Engineering; hyan@nps.navy.mil. Member AIAA.

†Professor, Beijing Institute of Control and Engineering.